Asymmetric D-braneworld

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In recent papers on the Randall-Sundrum D-braneworld model with Z_2 symmetry, it was shown that the effective gravity does not work as usual; that is, the gravity does not couple to the gauge field localized on the brane in the usual way. At first glance, there are two possibilities to avoid this serious problem. One is to remove the Z_2 symmetry and another is to consider a non-Bogomol'nyi-Prasad-Sommerfield (BPS) state. In this paper, we analyze the Randall-Sundrum D-braneworld model without Z_2 symmetry by long wave approximation. The result is an unexpected one; that is, the gauge field does not couple to the gravity on the brane in the leading order again. Therefore, the remaining possibility to recover the conventional gravitational theory would be non-BPS cases.

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I. INTRODUCTION

A new cosmological model, braneworld, based on a nonperturbative aspect of string theory, was proposed a few years ago [1]. The simplest model is the Randall-Sundrum (RS) model with warped compactification [2,3]. So far, it has been analyzed intensely in cosmological context because the self-gravity must be treated seriously and carefully. As a consequence, the RS model and its extensions are basically able to reproduce the standard cosmology. However, it is important to remember that the braneworld was inspired by the D-brane, which has many interesting characteristics which the RS model does not have. Therefore, we ask whether a more realistic braneworld model based on D-brane works as well. Recently, this issue was initiated in Ref. [4], in which Born-Infeld action, bulk gauge fields, and D-brane charge were appropriately taken into account using the type IIB supergravity compactified on S^5. See Refs. [5–8] for other issues based on D-brane.

A more tractable toy model has been investigated in Refs. [9–11]. There, the brane tension is assumed to equal the brane charge and Z_2 symmetry is imposed. Consequently, the gravity does not couple to the gauge fields at large distances although the gauge field is supposed to be localized on the brane. This is a serious problem if we want to use D-branes in a cosmological model. A possible solution to this problem was discussed in Ref. [12] assuming a non-Bogomol'nyi-Prasad-Sommerfield (BPS) state, that is, a brane with a charge different from the tension. As a result, it was shown that the gauge field may couple to the gravity and the gravitational constant is proportional to the cosmological constant on the brane.

In this paper, we address another case, which has two D-branes and does not have Z_2 symmetry (see Ref. [13] for an asymmetric braneworld model). It would be possible that the Z_2 symmetry induces the irregular behavior obtained in Refs. [4,9–11]. Therefore, we want to make clear the importance/unimportance of Z_2 symmetry. Surprisingly, our conclusion obtained in the above papers is unchanged if the two-form potentials for three-form fields are continuous at the branes. To discuss this issue, we employ the gradient expansion method [14]. Recently, it has been checked that such a method can give us the same result as that obtained by the linear perturbation at large distances [11].

The rest of this paper is organized as follows. In Sec. II, we describe the tractable toy model for D-branes. In Sec. III, we write down the field equations and the junction conditions. Then we solve the field equations under the junction conditions using the gradient expansion and derive the effective gravitational equation on the brane. In Sec. IV, we give summary and discussion. In the appendix, we sketch the conclusion obtained from the continuity of the two-form field potentials.

II. MODEL

We consider the asymmetric Randall-Sundrum two-brane model in type IIB supergravity compactified on S^5. The brane is described by Born-Infeld and Chern-Simons actions. So we begin with the following action (for example, see Refs. [4,9–11,15]):

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-\mathcal{G}}\Big[{}^{(5)}R - 2\Lambda - \frac{1}{2}|H|^2 - \frac{1}{2}(\nabla\chi)^2 - \frac{1}{2}|\tilde{F}|^2 - \frac{1}{2}|\tilde{G}|^2\Big] + S^{(+)}_{\rm brane} + S^{(+)}_{\rm CS} + S^{(-)}_{\rm brane} + S^{(-)}_{\rm CS}, \quad (1)$$

where $H_{MNK} = \frac{1}{2}\partial_{[M}B_{NK]}$, $F_{MNK} = \frac{1}{2}\partial_{[M}C_{NK]}$, $G_{K_1K_2K_3K_4K_5} = \frac{1}{4!}\partial_{[K_1}D_{K_2K_3K_4K_5]}$, $\tilde{F} = F + \chi H$, and $\tilde{G} = G + C\wedge H$. $M, N, K = 0, 1, 2, 3, 4$. B_{MN} and C_{MN} are two-form fields, and $D_{K_1K_2K_3K_4}$ is a four-form field. χ is a scalar field. $\mathcal{G}_{MN}$ is the metric of five-dimensional spacetime.

$S^{(\pm)}_{\rm brane}$ is given by Born-Infeld action

$$S^{(+)}_{\rm brane} = -\sigma \int d^4x \sqrt{-\det(h + \mathcal{F}^{(+)})}, \tag{2}$$

$$S^{(-)}_{\rm brane} = \sigma \int d^4x \sqrt{-\det(q + \mathcal{F}^{(-)})}, \tag{3}$$

where $h_{\mu\nu}$ and $q_{\mu\nu}$ are the induced metric on the $D_\pm$-brane and

$$\mathcal{F}^{(\pm)}_{\mu\nu} = B^{(\pm)}_{\mu\nu} + \sigma^{-1/2} F^{(\pm)}_{\mu\nu}. \tag{4}$$

$F_{\mu\nu}$ is the U(1) gauge field on the brane. $B_{\mu\nu}$ is the projection onto the brane of B_{MN}. Here $\mu, \nu = 0, 1, 2, 3$ and $\pm\sigma$ are $D_\pm$-brane tension. Hereafter, $\sigma > 0$ and then D_--brane has negative tension.

$S^{(\pm)}_{\rm CS}$ is Chern-Simons action

$$S^{(+)}_{\rm CS} = -\sigma \int d^4x \sqrt{-h} \epsilon^{\mu\nu\rho\sigma} \bigg[\frac{1}{4} \mathcal{F}^{(+)}_{\mu\nu} C^{(+)}_{\rho\sigma} + \frac{\chi}{8} \mathcal{F}^{(+)}_{\mu\nu} \mathcal{F}^{(+)}_{\rho\sigma} + \frac{1}{24} D^{(+)}_{\mu\nu\rho\sigma} \bigg], \tag{5}$$

$$S^{(-)}_{\rm CS} = \sigma \int d^4x \sqrt{-q} \epsilon^{\mu\nu\rho\sigma} \bigg[\frac{1}{4} \mathcal{F}^{(-)}_{\mu\nu} C^{(-)}_{\rho\sigma} + \frac{\chi}{8} \mathcal{F}^{(-)}_{\mu\nu} \mathcal{F}^{(-)}_{\rho\sigma} + \frac{1}{24} D^{(-)}_{\mu\nu\rho\sigma} \bigg]. \tag{6}$$

Here the brane charges are set equal to the brane tensions. Therefore, our model contains a BPS state of D-branes.

III. BASIC EQUATIONS

In this section we write down the basic equations and boundary conditions. Let us perform (1 + 4) decomposition along extra dimension

$$ds^2 = \mathcal{G}_{MN} dx^M dx^N = e^{2\phi(x)} dy^2 + g_{\mu\nu}(y, x) dx^\mu dx^\nu, \tag{7}$$

where y is the coordinate orthogonal to the brane. D_+-brane and D_--brane are supposed to locate at $y = y^{(+)} = 0$ and $y = y^{(-)} = y_0$.

The spacelike "evolutional" equations to the y direction are

$$e^{-\phi} \partial_y K = {}^{(4)}R - \kappa^2 \bigg({}^{(5)}T^\mu_\mu - \frac{4}{3} {}^{(5)}T^M_M \bigg) - K^2 - e^{-\phi} D^2 e^\phi, \tag{8}$$

$$e^{-\phi} \partial_y \tilde{K}^\mu_\nu = {}^{(4)}\tilde{R}^\mu_\nu - \kappa^2 \bigg({}^{(5)}T^\mu_\nu - \frac{1}{4} \delta^\mu_\nu {}^{(5)}T^\alpha_\alpha \bigg) - K \tilde{K}^\mu_\nu - e^{-\phi} [D^\mu D_\nu e^\phi]_{\rm traceless}, \tag{9}$$

$$\partial^2_y \chi + D^2 \chi + e^\phi K \partial_y \chi - \frac{1}{2} H_{y\alpha\beta} \tilde{F}^{y\alpha\beta} = 0, \tag{10}$$

$$\partial_y X^{y\mu\nu} + e^\phi K X^{y\mu\nu} + D_\alpha \phi H^{\alpha\mu\nu} + D_\alpha H^{\alpha\mu\nu} + \frac{1}{2} F_{y\alpha\beta} \tilde{G}^{y\alpha\beta\mu\nu} = 0, \tag{11}$$

$$\partial_y \tilde{F}^{y\mu\nu} + e^\phi K \tilde{F}^{y\mu\nu} + D_\alpha \phi \tilde{F}^{\alpha\mu\nu} + D_\alpha \tilde{F}^{\alpha\mu\nu} - \frac{1}{2} H_{y\alpha\beta} \tilde{G}^{y\alpha\beta\mu\nu} = 0, \tag{12}$$

$$\partial_y \tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = e^\phi K \tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{13}$$

where $X^{y\mu\nu} := H^{y\mu\nu} + \chi \tilde{F}^{y\mu\nu}$ and the energy-momentum tensor is

$$\kappa^{2(5)} T_{MN} = \frac{1}{2} \bigg[\nabla_M \chi \nabla_N \chi - \frac{1}{2} g_{MN} (\nabla \chi)^2 \bigg] + \frac{1}{4} [H_{MKL} H^{KL}_N - g_{MN} |H|^2] + \frac{1}{4} [\tilde{F}_{MKL} \tilde{F}^{KL}_N - g_{MN} |\tilde{F}|^2] + \frac{1}{96} \tilde{G}_{MK_1K_2K_3K_4} \tilde{G}^{K_1K_2K_3K_4}_N - \Lambda g_{MN}. \tag{14}$$

$K_{\mu\nu}$ is the extrinsic curvature, $K_{\mu\nu} = \frac{1}{2} e^{-\phi} \partial_y g_{\mu\nu}$. $\tilde{K}^\mu_\nu$ and ${}^{(4)}\tilde{R}^\mu_\nu$ are the traceless parts of K^μ_ν and ${}^{(4)}R^\mu_\nu$, respectively. Here D_μ is the covariant derivative with respect to $g_{\mu\nu}$.

The constraints on $y =$ const hypersurfaces are

$$-\frac{1}{2} \bigg[{}^{(4)}R - \frac{3}{4} K^2 + \tilde{K}^\mu_\nu \tilde{K}^\nu_\mu \bigg] = \kappa^{2(5)} T_{yy} e^{-2\phi}, \tag{15}$$

$$D_\nu K^\nu_\mu - D_\mu K = \kappa^{2(5)} T_{\mu y} e^{-\phi}, \tag{16}$$

$$D_\alpha (e^\phi X^{y\alpha\mu}) + \frac{1}{6} e^\phi F_{\alpha_1\alpha_2\alpha_3} \tilde{G}^{y\alpha_1\alpha_2\alpha_3\mu} = 0, \tag{17}$$

$$D_\alpha (e^\phi \tilde{F}^{y\alpha\mu}) - \frac{1}{6} e^\phi H_{\alpha_1\alpha_2\alpha_3} \tilde{G}^{y\alpha_1\alpha_2\alpha_3\mu} = 0, \tag{18}$$

$$D^\alpha (e^{-\phi} \tilde{G}_{y\alpha\mu_1\mu_2\mu_3}) = 0. \tag{19}$$

The junction conditions at the brane located $y = y^{(\pm)}$ are

$$\{K_{\mu\nu} - g_{\mu\nu} K\}^-_{y=y^{(\pm)}} = \kappa^2 \sigma (g_{\mu\nu} - T^{(\pm)}_{\mu\nu}) + O(T^2_{\mu\nu}), \tag{20}$$

$$\{H_{y\mu\nu}(y^{(\pm)}, x)\}^- = 2\kappa^2 \sigma e^\phi \mathcal{F}^{(\pm)}_{\mu\nu}, \tag{21}$$

$$\{\tilde{F}_{y\mu\nu}(y^{(\pm)}, x)\}^- = \kappa^2 \sigma e^\phi \epsilon_{\mu\nu\alpha\beta} \mathcal{F}^{(\pm)\alpha\beta}, \tag{22}$$

$$\{\tilde{G}_{y\mu\nu\alpha\beta}(y^{(\pm)},x)\}^- = 2\kappa^2\sigma e^{\phi}\epsilon_{\mu\nu\alpha\beta}, \quad (23)$$

$$\{\partial_y\chi(y^{(\pm)},x)\}^- = \frac{\kappa^2}{4}\sigma e^{\phi}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{(\pm)}_{\mu\nu}\mathcal{F}^{(\pm)}_{\alpha\beta}, \quad (24)$$

where, for any tensor field Q, $\{Q\}^-$ is defined as $\{Q\}^- \equiv Q_R - Q_L$. Subscripts R and L denote the quantity evaluated on the right-hand and the left-hand side of the D_+ brane, respectively. In the above,

$$T^{(\pm)\mu}{}_{\nu} = \mathcal{F}^{(\pm)\mu\alpha}\mathcal{F}^{(\pm)}_{\nu\alpha} - \frac{1}{4}\delta^{\mu}_{\nu}\mathcal{F}^{(\pm)}_{\alpha\beta}\mathcal{F}^{(\pm)\alpha\beta}. \quad (25)$$

From the junction condition for χ, we can omit the contribution of χ to the gravitational equation on the brane in the approximations which we will employ. Moreover, we omit the quadratic term in Eq. (20).

IV. EFFECTIVE THEORY

In this section, we approximately solve the bulk field equations by long wave approximation (gradient expansion [14]) and derive the effective gravitational theory on the brane.

In the case with bulk fields, we must carefully use the geometrical projection method [13,16] because the projected Weyl tensor $E_{\mu\nu}$ contains the leading effect from the bulk fields.

The bulk metric is written again as

$$ds^2 = e^{2\phi(x)}dy^2 + g_{\mu\nu}(y,x)dx^{\mu}dx^{\nu}. \quad (26)$$

The induced metric on the brane will be denoted by $h_{\mu\nu} := g_{\mu\nu}(0,x)$ and then

$$g_{\mu\nu}(y,x) = a^2(y,x)[h_{\mu\nu}(x) + \overset{(1)}{g}_{\mu\nu}(y,x) + \cdots]. \quad (27)$$

In the above $\overset{(1)}{g}_{\mu\nu}(0,x) = 0$ and $a(0,x) = 1$. In a similar way, the extrinsic curvature is expanded as

$$K^{\mu}_{\nu} = \overset{(0)}{K}{}^{\mu}_{\nu} + \overset{(1)}{K}{}^{\mu}_{\nu} + \overset{(2)}{K}{}^{\mu}_{\nu} + \cdots. \quad (28)$$

The small parameter is $\epsilon = (\ell/L)^2 \ll 1$, where L and ℓ are the curvature scale on the brane and the bulk anti-de Sitter curvature scale, respectively.

A. Background

Without derivation we present the background spacetime. It is locally five-dimensional anti-de Sitter-like spacetime

$$ds^2 = e^{2\phi(x)}dy^2 + a^2(y,x)h_{\mu\nu}dx^{\mu}dx^{\nu}, \quad (29)$$

where

$$a = a_R = e^{-(y/\ell_R)e^{\phi}} \quad \text{for } 0 \le y \le y_0 = y^{(-)} \quad (30)$$

and

$$a = a_L = e^{\{(y/\ell_L)-[(1/\ell_R)+(1/\ell_L)]y_0\}e^{\phi}}$$
$$\text{for } y_0 \le y \le y_* = \left(1 + \frac{\ell_L}{\ell_R}\right)y_0. \quad (31)$$

The $y = y_*$ hypersurface is identified with the $y = 0$ hypersurface.

The junction conditions for the extrinsic curvature are

$$\overset{(0)}{K}{}^{R}_{\mu\nu} - \overset{(0)}{K}{}^{L}_{\mu\nu} = -\left(\frac{1}{\ell_R} + \frac{1}{\ell_L}\right)h_{\mu\nu} = -\frac{\kappa^2}{3}\sigma h_{\mu\nu} \quad (32)$$

and

$$\frac{\kappa^2}{2}[\overset{(0)}{K}{}^{R}_{\mu\nu} + \overset{(0)}{K}{}^{L}_{\mu\nu}](-\sigma g^{\mu\nu}) = \{{}^{(5)}G_{\mu\nu}n^{\mu}n^{\nu}\}^-$$
$$= (\Lambda^{\rm tot}_R - \Lambda^{\rm tot}_L), \quad (33)$$

where

$$\Lambda^{\rm tot}_{R,L} = \Lambda_{R,L} - \tfrac{1}{96}\tilde{G}^{R,L}_{y\alpha_1\alpha_2\alpha_3\alpha_4}\tilde{G}^{y\alpha_1\alpha_2\alpha_3\alpha_4}_{R,L}. \quad (34)$$

Equations (32) and (33) become

$$\frac{1}{\ell_R} + \frac{1}{\ell_L} = \frac{\kappa^2}{3}\sigma \quad (35)$$

and

$$2\kappa^2\sigma\left(-\frac{1}{\ell_R} + \frac{1}{\ell_L}\right) = \Lambda^{\rm tot}_R - \Lambda^{\rm tot}_L. \quad (36)$$

The junction condition for $\tilde{G}_5$ is

$$\{\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}(y^{(\pm)},x)\}^- = 2\kappa^2\sigma e^{\phi}\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}. \quad (37)$$

Since the solutions are given by

$$\tilde{G}^{\,R}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = \alpha_R a_R^4 e^{\phi}\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4} \quad (38)$$

and

$$\tilde{G}^{\,L}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = \alpha_L a_L^4 e^{\phi}\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}, \quad (39)$$

the junction condition becomes

$$\{\alpha\}^- = 2\kappa^2\sigma. \quad (40)$$

Then the continuity conditions for a four-form potential of $\tilde{G}_5$ completely fix the value of α^R and α^L. To see this, we begin with

$$\tilde{G}^{\,R,L}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = \partial_y D^{R,L}_{\alpha_1\alpha_2\alpha_3\alpha_4}. \quad (41)$$

Then the potential $D^{R,L}_{\alpha_1\alpha_2\alpha_3\alpha_4}$ can be solved as

$$D^{R,L}_{\alpha_1\alpha_2\alpha_3\alpha_4} = \mp\ell_{R,L}a^4_{R,L}\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4} + d^{R,L}_{\alpha_1\alpha_2\alpha_3\alpha_4}(x), \quad (42)$$

where $d^{R,L}_{\alpha_1\alpha_2\alpha_3\alpha_4}(x)$ is the constant of integration. Here it is natural to assume the continuity $D^{R,L}_{\alpha_1\alpha_2\alpha_3\alpha_4}$ on branes; that is,

$$D^{R}_{\alpha_1\alpha_2\alpha_3\alpha_4}(y^{(\pm)},x) = D^{L}_{\alpha_1\alpha_2\alpha_3\alpha_4}(y^{(\pm)},x). \quad (43)$$

Then we obtain

$$\{d_{\alpha_1\alpha_2\alpha_3\alpha_4}\}^- = \tfrac{1}{4}(\ell_L\alpha_L + \ell_R\alpha_R)\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4} \tag{44}$$

and

$$\{d_{\alpha_1\alpha_2\alpha_3\alpha_4}\}^- = \tfrac{1}{4}(\ell_L\alpha_L + \ell_R\alpha_R)a_0^4\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{45}$$

where $a_0 = a_R(y = y_0) = e^{-(y_0/\ell_R)e^\phi}$. The above equations give us

$$\frac{\alpha_R}{\alpha_L} = \frac{\frac{1}{\ell_R}}{-\frac{1}{\ell_L}}. \tag{46}$$

Together with Eq. (40), then, we get the results

$$\alpha_R = \frac{6}{\ell_R} \quad \text{and} \quad \alpha_L = -\frac{6}{\ell_L}. \tag{47}$$

Hereafter, we set RS tuning

$$\Lambda^{\rm tot}_{R,L} + \frac{6}{\ell^2_{R,L}} = \Lambda_{R,L} + \frac{15}{\ell^2_{R,L}} = 0. \tag{48}$$

It means that the net cosmological constant on the brane vanishes.

B. Form fields

Let us focus on H_3 and $\tilde{F}_3$ fields, which can contribute to the gravity on the brane. From first order differential Eqs. (11) and (12) for them we obtain the following second order differential equation:

$$\partial_y^2 H^{R,L}_{y\mu\nu} - \frac{36}{\ell^2_{R,L}}e^{2\phi}H^{R,L}_{y\mu\nu} = 0. \tag{49}$$

The solution is given by

$$H^{R,L}_{y\mu\nu} = a^{-6}_{R,L}\alpha^{R,L}_{\mu\nu} + a^6_{R,L}\beta^{R,L}_{\mu\nu} \tag{50}$$

and

$$\begin{aligned}\tilde{F}^{R,L}_{y\mu\nu} &= \pm\frac{\ell_{R,L}}{12}\epsilon_{\mu\nu}{}^{\alpha\beta}\partial_y H^{R,L}_{y\alpha\beta} \\ &= \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}(a^{-6}_{R,L}\alpha^{R,L}_{\mu\nu} - a^6_{R,L}\beta^{R,L}_{\mu\nu}).\end{aligned} \tag{51}$$

The junction condition at $y = 0$ implies

$$\{\alpha_{\mu\nu}\}^- = 2\kappa^2\sigma e^\phi\mathcal{F}^{(+)}_{\mu\nu} \tag{52}$$

and

$$\{\beta_{\mu\nu}\}^- = 0. \tag{53}$$

In the same way, the junction conditions on $y = y_0$ provide us

$$a_0^{-6}\{\alpha_{\mu\nu}\}^- = 2\kappa^2\sigma e^\phi\mathcal{F}^{(-)}_{\mu\nu} \tag{54}$$

and

$$\{\beta_{\mu\nu}\}^- = 0. \tag{55}$$

Then Eqs. (52) and (54) lead us to

$$\mathcal{F}^{(+)}_{\mu\nu} = a_0^6\mathcal{F}^{(-)}_{\mu\nu}. \tag{56}$$

Finally, the continuity for the potential B_2 and C_2 of H_3 and F_3 determines $\alpha^{R,L}_{\mu\nu}$ and $\beta^{R,L}_{\mu\nu}$ as

$$\alpha^{R,L}_{\mu\nu} = \pm\frac{6}{\ell_{R,L}}e^\phi\mathcal{F}^{(+)}_{\mu\nu} \tag{57}$$

and

$$\beta^{R,L}_{\mu\nu} = 0. \tag{58}$$

See the appendix for the details of the argument on continuity of B_2.

As a consequence, we can uniquely determine $H_{y\mu\nu}$ and $\tilde{F}_{y\mu\nu}$

$$H^{R,L}_{y\mu\nu} = \pm\frac{6}{\ell_{R,L}}e^\phi a^{-6}_{R,L}\mathcal{F}^{(+)}_{\mu\nu} \tag{59}$$

and

$$\tilde{F}^{R,L}_{y\mu\nu} = \pm\frac{3}{\ell_{R,L}}e^\phi a^{-6}_{R,L}\epsilon_{\mu\nu}{}^{\alpha\beta}\mathcal{F}^{(+)}_{\alpha\beta}. \tag{60}$$

C. Extrinsic curvature and effective theory

We are now ready to derive the effective theory on the brane. To do so, we will solve the evolutional equation of extrinsic curvature [Eq. (9)]. The solution of the traceless part is

$$\begin{aligned}\overset{(1)}{\tilde{K}}{}^{R\mu}_{\nu} &= -\frac{\ell_R}{2a_R^2}{}^{(4)}\tilde{R}^\mu_\nu - \frac{3}{\ell_R}a_R^{-16}T^{(+)\mu}_\nu + \frac{\chi^{\mu R}_\nu}{a_R^4} \\ &\quad - \frac{1}{a_R^2}\Big[\mathcal{D}^\mu\mathcal{D}_\nu d_R - \frac{1}{\ell_R}\mathcal{D}^\mu d_R\mathcal{D}_\nu d_R\Big]_{\rm traceless}\end{aligned} \tag{61}$$

and

$$\begin{aligned}\overset{(1)}{\tilde{K}}{}^{L\mu}_{\nu} &= \frac{\ell_L}{2a_L^2}{}^{(4)}\tilde{R}^\mu_\nu + \frac{3}{\ell_L}a_L^{-16}T^{(+)\mu}_\nu + \frac{\chi^{\mu L}_\nu}{a_L^4} \\ &\quad + \frac{1}{a_L^2}\Big[\mathcal{D}^\mu\mathcal{D}_\nu d_L - \frac{1}{\ell_L}\mathcal{D}^\mu d_L\mathcal{D}_\nu d_L\Big]_{\rm traceless},\end{aligned} \tag{62}$$

where $\chi^{R,L}_{\mu\nu}$ are the constants of integration and $d \equiv ye^\phi$ is the proper distance between the two D-branes, which is called radion. $\mathcal{D}_\mu$ is the covariant derivative with respect to $h_{\mu\nu}$. The junction condition for $\tilde{K}^\mu_\nu$ at $y = 0$ and the above solution give us

$$\begin{aligned}-\kappa^2\sigma T^{(+)\mu}_\nu &= \{\overset{(1)}{\tilde{K}}{}^\mu_\nu\}^- \\ &= -\frac{1}{2}(\ell_R + \ell_L){}^{(4)}\tilde{R}^\mu_\nu - 3\Big(\frac{1}{\ell_R} + \frac{1}{\ell_L}\Big)T^{(+)\mu}_\nu \\ &\quad + \{\chi^\mu_\nu\}^-.\end{aligned} \tag{63}$$

The stress tensor in the left-hand side is exactly canceled out with that in the right-hand side. Thus, the gravitational equation becomes

$$ {}^{(4)}G_{\mu\nu}(h) = \frac{2}{\ell_R + \ell_L}\{\chi_{\mu\nu}\}^-. \tag{64} $$

$\{\chi_{\mu\nu}\}^-$ can be determined by the remaining junction condition at $y = y_0$. From the junction condition, indeed, we first obtain

$$ \begin{aligned} -\kappa^2\sigma T_\nu^{(-)\mu} = & -\frac{1}{2}a_0^{-2}(\ell_R + \ell_L){}^{(4)}\tilde{R}_\nu^\mu \\ & - 3\left(\frac{1}{\ell_R} + \frac{1}{\ell_L}\right)a_0^{-16}T_\nu^{(+)\mu} + a_0^{-4}\{\chi_\nu^\mu\}^- \\ & - a_0^{-2}\left(1 + \frac{\ell_L}{\ell_R}\right)\Big[\mathcal{D}^\mu\mathcal{D}_\nu d_R^0 \\ & - \frac{1}{\ell_R}\mathcal{D}^\mu d_R^0\mathcal{D}_\nu d_R^0\Big]_{\text{traceless}}, \end{aligned} \tag{65} $$

where $d_R^0 = d_R(y = y_0)$ and we used the fact $d_L^0 = (\ell_L/\ell_R)d_R^0$. In the same way with the argument at $y = 0$ brane, using Eq. (56), we can show the left-hand side is exactly canceled with the second term in the right-hand side. Finally, Eqs. (64) and (65) can be summarized as

$$ \begin{aligned} {}^{(4)}G_{\mu\nu}(h) = & \frac{2}{\ell_R(a_0^{-2} - 1)}\Big[\mathcal{D}_\mu\mathcal{D}_\nu d_R \\ & - \frac{1}{\ell_R}\mathcal{D}_\mu d_R\mathcal{D}_\nu d_R\Big]_{\text{traceless}}. \end{aligned} \tag{66} $$

This is our main result.

The equation for radion d_R^0 can be derived from the trace part of the extrinsic curvature and then

$$ \mathcal{D}^2 d_R - \frac{1}{\ell_R}(\mathcal{D}d_R)^2 = 0. \tag{67} $$

V. SUMMARY AND DISCUSSION

In this paper we investigated a D-braneworld model without Z_2 symmetry and derived the effective theory on the D-brane. Surprisingly, the gauge fields do not couple to the gravity on the D-brane at large distances. The result is basically the same as that in the previous works [4,9–11].

Thus, the remaining possibility to recover the conventional gravitational theory in D-braneworld would be non-BPS cases. In a non-BPS state, a nonzero cosmological constant appears on the brane. In the one-D-brane model discussed Ref. [12], indeed, the appearance of the gravitational coupling to the gauge field localized on the brane was confirmed using the gradient expansion method. Moreover, it turned out that the gravitational constant is proportional to the cosmological constant on the brane. Therefore, the presence of the cosmological constant on the branes seems to be the only solution to the D-braneworld model if B_2, C_2, and D_4 are continuous at the branes.

There is another possibility: B_2, C_2, D_4 are not continuous at the brane due to some interaction terms [17]. We should consider this possibility in future work.

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APPENDIX: CONTINUITY FOR B_2 AND C_2

The potentials B_2 and C_2 have the following solutions:

$$ B_{\mu\nu}^{R,L} = \mp\frac{\ell_{R,L}}{6}[a_{R,L}^{-6}\alpha_{\mu\nu}^{R,L}(x) - a_{R,L}^6\beta_{\mu\nu}^{R,L}(x)] + b_{\mu\nu}^{R,L}(x) \tag{A1} $$

and

$$ \begin{aligned} C_{\mu\nu}^{R,L} = & \mp\frac{\ell_{R,L}}{12}\epsilon_{\mu\nu}{}^{\alpha\beta}[a_{R,L}^{-6}\alpha_{\alpha\beta}^{R,L}(x) + a_{R,L}^6\beta_{\alpha\beta}^{R,L}(x)] \\ & + c_{\mu\nu}^{R,L}(x), \end{aligned} \tag{A2} $$

where $b_{\mu\nu}^{R,L}(x)$ and $c_{\mu\nu}^{R,L}(x)$ are the constants of integration. From the continuity at $y = y^{(\pm)}$, it is easy to obtain

$$ \ell_R\alpha_{\mu\nu}^R + \ell_L\alpha_{\mu\nu}^L = 0 \tag{A3} $$

and

$$ \ell_R\beta_{\mu\nu}^R + \ell_L\beta_{\mu\nu}^L = 0. \tag{A4} $$

Together with Eqs. (52) and (53), we see

$$ \alpha_{\mu\nu}^{R,L} = \mp\frac{6}{\ell_{R,L}}e^\phi\mathcal{F}_{\mu\nu}^{(+)} \tag{A5} $$

and

$$ \beta_{\mu\nu}^{R,L} = 0. \tag{A6} $$

These results are what we wanted to show.

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